Understanding Frequency Accuracy in Crystal Controlled Instruments

Application Note
Understanding Frequency Accuracy in Crystal Controlled Instruments:

Synthesizers, vector network analyzers, frequency counters, spectrum analyzers and many other instruments must accurately create or measure frequency. However, their accuracy is affected by time, temperature, frequency and even gravity. This paper explains these effects in understandable terms and explains how to determine absolute frequency accuracy. This paper is not a thesis on crystal time base design; rather, it explains the external specifications of crystal time bases and how these specifications affect instrument performance.

A frequency generation or measurement instrument must contain an internal frequency standard to which everything is referenced. It is called the time base standard. This standard is invariably a quartz crystal oscillator. The relative accuracy, size and cost of quartz crystal oscillators make them the time base standards of choice for instrumentation.

Overview of How Quartz Crystal Oscillators Work:

While this paper does not delve deeply into quartz crystal oscillator design, a basic understanding of how quartz oscillates is essential to understanding the associated uncertainties. Quartz crystal is a piezo-electric substance. Piezo-electricity is a phenomenon where a mechanical stress on the crystal causes a voltage across the crystal. Conversely, a voltage applied across the crystal will cause mechanical movement within the crystal. If an AC voltage is applied across the crystal, the crystal will begin to vibrate. A given crystal will have a rate at which it vibrates best. This rate is the resonant frequency of the crystal and is determined by the cut, size, and shape of the crystal, as well as any fixed mechanical stress applied to the crystal. If this crystal is used as the tuning element in an oscillator circuit, the circuit will oscillate at this resonant frequency.

Quartz is used because a given crystal will have a very narrow range of frequencies at which it will resonate. If everything is made correctly, this can result in an oscillator frequency with very little uncertainty, obviously a highly desirable characteristic when building a frequency standard. The points to note are that the cut, size and shape of the crystal affect the resonant frequency. Likewise, mechanical stress on the crystal will affect the resonant frequency.

UNCERTAINTY VS. ERROR

Although error and uncertainty are often used interchangeably, they are not the same.

Error defines how far the actual frequency is from the expected frequency. For example, if the expected frequency is 10.000 GHz and the actual frequency is 10.001 GHz, the error is +0.001 GHz. Knowing an error implies that you know the actual frequency to a high degree of accuracy.

Uncertainty defines a range within which the frequency must be. For example, if the expected frequency is 10.000 GHz and the uncertainty is ±0.001 GHz, then the actual frequency is between 9.999 GHz and 10.001 GHz. You don't precisely know the actual frequency but you know it must be between these two values.

Fractional Uncertainty vs. Frequency Uncertainty

Time base uncertainties are expressed in fractional parts, such as 5x10^-9 (five parts in ten to the minus ninth). Determining actual frequency uncertainty from fractional uncertainty is simply a matter of multiplying the frequency by the fraction. For example, if a synthesizer is producing 10 GHz, and the time base uncertainty is determined to be 5x10^-8, the uncertainty is:

\[(10 \text{ GHz}) \times (5x10^{-9}) = (1x10^{10} \text{ Hz}) \times (5x10^{-9}) = 5x10^2 \text{ Hz} = 500 \text{ Hz}\]

So, the absolute frequency uncertainty in this example is 500 Hz.

(Note: The time base uncertainties presented here are discussed as fractional uncertainty, then tied to absolute frequency uncertainty at the end of the paper.)

Time Base Uncertainties

Time base uncertainties fall into the following categories:

1) Temperature effects
2) Uncertainty with respect to time
   a) Short-term stability
   b) Long-term Aging
3) Mechanical effects
   a) Shock and vibration
   b) Gravity
**Temperature effects**

Temperature changes cause size changes in mechanical devices. In a quartz crystal, size changes cause frequency changes. Therefore, temperature changes cause frequency changes.

There are three methods employed by crystal time base manufacturers to account for temperature:

1) Ignore temperature
2) Compensate for temperature
3) Control the temperature

The simplest time bases, usually referred to as Room Temperature Crystal Oscillators (RTXO), do not try to compensate for temperature. They are used when only moderate accuracy is required and when the equipment is not expected to endure significant temperature changes. With an RTXO, uncertainty with respect to temperature is the most significant source of uncertainty. RTXO temperature stability is typically ±1x10⁻⁵ over 0 to 70°C.

Temperature Compensated Crystal Oscillators (TCXO) measure the ambient temperature and adjust the oscillator to a calibrated compensation curve. This compensation makes TCXOs more stable with temperature variations than RTXOs, but it does not make them immune. TCXO temperature stability is typically ±5x10⁻⁷ over 0 to 70°C.

The most stable crystal time bases with respect to temperature are OCXOs (Oven Controlled Crystal Oscillator). In an OCXO the crystal is inside of an oven that holds the temperature constant, independent of the ambient temperature. With the temperature held constant, the crystal can be optimized for other parameters, such as aging rate. So, besides better temperature stability, OCXOs typically have better aging characteristics than TCXOs or RTXOs. Most time base controlled test equipment offer OCXOs either as standard equipment or as optional equipment. OCXO temperature stability is typically ±5x10⁻¹⁰ over 0 to 70°C.

**Temperature uncertainties are specified in one of two ways:**

a) Fractional uncertainty per °C, for example ±5x10⁻¹⁰/°C.

b) Total fractional uncertainty over a specified temperature range, for example ±5x10⁻⁷ over 0-50°C.

If an oscillator is specified as fractional uncertainty per °C, you determine uncertainty simply by determining the difference between the ambient temperature and 25°C, and multiplying that difference by the fractional uncertainty. For example:

Spec temperature stability for OCXO: ±5x10⁻¹⁰/°C

Ambient temperature: 21°C

25°C - 21°C = 3°C

3°C x ±5x10⁻¹⁰/°C = ±15x10⁻¹⁰ = ±1.5x10⁻⁹

The uncertainty is plus-or-minus because we do not know if the time base will drift up or down with respect to temperature. Note that the reference temperature is usually not specifically stated. When the reference temperature is not specifically stated, 25°C should be used.
If an oscillator is specified as total fractional uncertainty over a specified temperature range, that uncertainty must be used at any temperature. The uncertainty cannot be interpolated to error per °C. For example, for an oscillator that is specified as 7x10^{-7} over 0-70°C, the uncertainty is NOT 1x10^{-8}/°C. The maximum error could occur at 70°C or at 25°C. Typically, crystal oscillators are designed to have the least change near 25°C and permitted to have greater change at extreme temperatures, but this is not guaranteed. So without knowledge of the specific temperature curve, if an oscillator is specified as total fractional uncertainty over a specified temperature range the worst case uncertainty must be assumed at any temperature.

**Uncertainty with Respect to Time**

Uncertainty with respect to time falls into three categories, depending on the rate of change of the crystal frequency. Single-sideband (SSB) phase noise defines frequency variations that occur at a fast rate, usually <0.1 second. Short-term stability defines uncertainties <1 second, and long-term aging defines uncertainties >1 second, extending to months or years.

Long-term aging is invariably the most significant uncertainty. SSB phase noise and short-term stability may or may not have significance depending on the application. For example, in Anritsu's El Toro family of microwave synthesizers, the SSB phase noise of the standard time base is very important because time base noise can be multiplied by as much as 60 dB (for a 100 GHz output). However, the SSB phase noise of the optional high-stability time base is not significant because the synthesizer locks to it with a very narrow phase locked loop that ignores any variations faster than 1 second.

**SSB Phase Noise**

Although SSB phase noise is a measure of variation vs. time (time domain), it is specified as a frequency domain measurement. A typical SSB phase noise spec would be <-140 dBc/Hz at 1 kHz offset from 10 MHz. This means that for a 10 MHz time base, if you measure the noise amplitude 1 kHz away from the time base frequency, using a measurement device with a 1 Hz bandwidth, the noise amplitude will be 140 dB or more below the amplitude of the time base. SSB phase noise, when provided, is usually a graph showing dBc/Hz for offsets from 10 Hz to 100 kHz.

Usually, in an instrument the SSB phase noise of the time base is not relevant to the user and is not provided. Typically, the effects of time base phase noise on the instrument are reflected in the specs. For example, a synthesizer's SSB phase noise is typically provided.

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**Short-term Stability**

Short-term stability is any effect faster than long-term aging. Long-term aging is, of course, any effect slower than short-term stability. If you think this is ambiguous, you're right. Fortunately, specifications for short-term stability have been pseudo-standardized as the RMS uncertainty in the time base, averaged over 1 second. It is primarily caused by noise in the active circuitry in the oscillator.

Short-term stability is a random effect that can be effectively removed by averaging or by integrating over time. It is only a factor when measurements are significantly less than a second.

Note that short-term stability is specified as an RMS value. This means that the uncertainty due to short-term stability is not known at any instant in time. All we know is that uncertainty due to short-term stability will be less than spec most of the time.

Short-term stability is a traditional spec that is often omitted in modern time bases because modern circuitry has reduced short-term stability uncertainty to a very small number. A typical short-term stability specification for a quality time base is 1x10^{-10} RMS over 1 second.
**Long-term Aging**

Time base aging causes the most consternation when discussing frequency accuracy because the phenomenon of aging prevents us from attaching a fixed value to uncertainty. The uncertainty changes continuously. It is not random, yet it is not constant. However time base aging is simply a fact of life; Crystal oscillator frequencies change with time. This leads to a very important consideration:

In other words, to know the uncertainty of a time base controlled instrument, not only must you know the time base specifications but you must also know how much time has elapsed since the time base was calibrated.

**What Causes Aging?**

Aging primarily comes from 3 factors:
1) Relaxation of Mechanical Stress
2) Movement of Impurities
3) Material comes loose from, or adheres to, the crystal

**Relaxation of Mechanical Stress**

When a crystal is mounted into a mounting structure, mechanical stress is necessarily introduced into the crystal. For example, one point on the crystal may be held a bit tighter than another point. Over time, as the mounting structure relaxes, this stress changes. The changing mechanical stress results in changing oscillator frequency, or aging.

It is primarily because of this changing stress effect that a second effect must be considered:

**Material comes loose from, or adheres to, the crystal**

As the crystal sits and vibrates, microscopic pieces of dust are thrown off in the crystal. (It is also possible that microscopic pieces of dust can be attracted to the crystal.) This change in mass causes changes in frequency. The dust may be different sizes, leading to another fact of aging:

**Old Crystals Age Less**

Note that the causes of aging will reduce over time. Mechanical stresses will approach an equilibrium. Impurities will find a location to settle, etc. This leads to a fourth fact of aging:

**Time Base Error**

Time Base error measured over 1 year. Note that aging rate, equivalent to the slope of the curve, reduces with time.

Even if your instrument has been sitting on a shelf in the back of a warehouse, aging has still been occurring and uncertainty is not the same as when it was placed on the shelf. 'Shelf time' must always be considered when determining uncertainties due to long-term aging.

**Movement of Impurities**

Quartz crystals have minute amounts of impurities, either in the quartz, or introduced in the handling. As time goes by, these impurities will migrate through the crystal, causing aging.
Specifying aging rate per month or per year is quite practical for defining instrument accuracy. It permits the instrument manufacturer to guarantee better long-term performance. However, it can cause problems when testing. An aging rate that is specified per year, can not be properly tested in a few days. A proper way to test 'per year' aging on a time base is to note time base change over yearly calibration cycles.

Time base aging rate varies from about $1 \times 10^{-7}$/month for a typical TCXO to $1 \times 10^{-10}$/day for a very high quality OCXO.

**Mechanical Shock and Vibration**

As discussed previously, physical stress on a quartz crystal causes oscillator frequency changes. This physical stress can result from movement of the instrument, such as vibration. Vibration and shock are transient effects and usually can be ignored. However, in a moving environment, such as aircraft or water craft, they must be considered—particularly as to how they affect short-term stability and phase noise.

**Gravity**

Gravity applies a mechanical stress to the crystal, affecting the frequency. If the instrument is always oriented the same, gravity effects do not matter. However, if the orientation of the instrument changes between measurements, gravity will have an effect.

Although gravity is not usually a significant effect, it must be considered in high gravitational conditions, such as acceleration. Note that the effects of gravity are different from those of shock and vibration. This is because shock and vibration can be reduced with mechanical damping components, whereas gravity is a constant effect that can not be removed.

Time bases typically are sensitive to gravity at about $1 \times 10^{-9}/g$.

**Most Significant Sources of Error**

The most significant uncertainty in a TCXO is temperature. Typically, $>50\%$ of the uncertainty in a TCXO is due to temperature. The second most significant uncertainty in a TCXO is long-term aging (assuming 1 year calibration cycles) at about 40%.

In an OCXO, the most significant uncertainty is long-term aging at about 90%. Since long-term aging accounts for such a high percentage of the uncertainty, in many requirements, it can be considered the only significant uncertainty. This greatly simplifies uncertainty calculations.

**Determining Total Uncertainty**

Total uncertainty is the sum of all the individual uncertainties. Time base errors may cause the time base frequency to change in different directions, partially canceling each other. However, without knowledge of the exact results of specific effects, the only way to consider total uncertainty is the sum total of all the individual causes.
Examples

Example 1:
A typical TCXO time base has the following specifications:

Temperature stability: $<1 \times 10^{-6}$ over 0°C to 50°C
Short-term stability: Not specified, assume $1 \times 10^{-10}$ RMS/second
Aging rate: $<1 \times 10^{-7}$/month
Shock and Vibration: Not specified, assume $1 \times 10^{-10}$/g
Gravity: Not specified, assume $1 \times 10^{-9}$/g

Assume the instrument is a frequency counter in a laboratory environment. The ambient temperature is 25°C and it has been six months since calibration.

Fractional uncertainties:
- Temperature: $1 \times 10^{-6}$
- Short-Term Stability: $1 \times 10^{-10}$
- Aging: $6$ months x $(1 \times 10^{-7}$/month) = $6 \times 10^{-7}$
- Vibration: N/A
- Gravity: N/A

Total Fractional Uncertainty: $1 \times 10^{-6} + 1 \times 10^{-10} + 6 \times 10^{-7}$
  = $1.6001 \times 10^{-6}$
  = $1.6 \times 10^{-6}$

Uncertainty at 10 GHz: $10$ GHz x $1.6 \times 10^{-6}$
  = $1 \times 10^{10}$ x $1.6 \times 10^{-6}$
  = $1.6 \times 10^{4}$
  = 10 kHz

Note that the sensitivity of a TCXO to temperature significantly hampers the ability to test aging rate. Even very small changes in temperature can cause significant frequency changes. The only way to accurately measure aging rate on a TCXO is to measure it over a very long period of time in a temperature controlled environment.

Example 2:
The high stability time base in an Anritsu El Toro synthesizer has the following specifications:

Temperature stability: $<2 \times 10^{-10}$/°C over 0°C to 55°C
Short-term stability: Not specified, assume $1 \times 10^{-10}$ RMS/second
Aging rate: $<5 \times 10^{-10}$/day
Shock and Vibration: Not specified, assume $1 \times 10^{-10}$/g
Gravity: Not specified, assume $1 \times 10^{-9}$/g

Assume that the instrument is an Anritsu El Toro synthesizer in an aircraft. The output frequency is 10 GHz. The ambient temperature is 20°C. The instrument is subjected to 5g peak vibration. It has been six months since the last calibration.

Fractional uncertainties:
- Temperature: $(25°C - 20°C) \times (2 \times 10^{-10}$/°C)
  = $5°C \times (2 \times 10^{-10}$/°C)
  = $1 \times 10^{-9}$
- Short-Term Stability: $1 \times 10^{-10}$
- Aging: $6$ months x $5 \times 10^{-10}$/day
  = $182$ days x $5 \times 10^{-10}$/day
  = $9.1 \times 10^{-8}$
- Vibration: $5g \times 1 \times 10^{-10}$/g = $5 \times 10^{-10}$
- Gravity: N/A

Total Fractional Uncertainty: $1 \times 10^{-9} + 1 \times 10^{-10} + 9.1 \times 10^{-8} + 5 \times 10^{-10}$
  = $9.26 \times 10^{-8}$

Uncertainty at 10 GHz: $10$ GHz x $9.26 \times 10^{-8}$
  = $1 \times 10^{10}$ Hz x $9.26 \times 10^{-8}$
  = $9.26 \times 10^{2}$
  = 926 Hz

Uncertainty at 10 GHz due to aging:
$10$ GHz x $9.1 \times 10^{-8}$
  = $1 \times 10^{10}$ Hz x $9.1 \times 10^{-8}$
  = $9.1 \times 10^{2}$
  = 910 Hz

Note that in contrast to the TCXO, the aging rate uncertainty in an OCXO is the majority of the total uncertainty. In fact, it is so much of the total uncertainty that:

**In an OCXO based instrument, the other uncertainties can often be ignored and the aging rate can provide a close approximation of the total uncertainty.**
Summary

Many instruments rely on quartz crystal time bases for frequency accuracy and stability. Quartz crystal time bases display uncertainty characteristics that are dependent on temperature, time, and mechanical stress. Understanding the performance specifications of an instrument's time base is not sufficient to determine uncertainty. The amount of time since the instrument was calibrated and, possibly, the ambient environmental conditions, must also be known to determine uncertainty.